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Report on the dissertation entitled "Algorithms For Quadratic Forms Over Global Function Fields" by Mawunyo Kofi Darkey-Mensah under supervision of Dr. hab. Przemysław Koprowski.

The main goal of this dissertation is to present computational tools for some problems in algebra. Most of the algorithms presented in it are related to quadratic forms over a global field (of characteristic different than 2). This is related to the problem of the factorization of fractional ideals in the ring of polynomial functions of a global function field. This dissertation also presents algorithms for the latter problem.

In the introduction, 7 problems are presented. Each of them has its importance in algebra. The dissertation presents 22 computational algorithms to solve these problems. In order to prove the correctness of these algorithms, a broad knowledge in many fields of algebra is required. The mathematical background to solve these problems is presented in Chapter 2.

Chapter 3 is devoted to present algorithms for the factorization of ideals. The algorithms 1-3 present different ways to factorize some types of ideals. Finally, Algorithm 4 combines the previous algorithms to present a full factorization of ideals.

In Chapter 4 many invariants of quadratic forms are calculated. These invariants are the isotropy, hyperbolicity, Witt index and similarity. For each of these invariants the author presents the corresponding algorithms (Algorithms 5–13).

Chapter 5 is devoted to compute the anisotropic part of a quadratic formula. The Witt decomposition Theorem guarantees that every non-degenerate quadratic form can be decomposed as the orthogonal sum of an anisotropic and a hyperbolic form. Algorithms 14–17 are computational methods to calculate the anisotropic part of a quadratic form in specific cases.

Finally, in Chapter 6 the author presents algorithms to calculate the length of a sum of squares. This number is the smallest length of a sum of squares of a given element in a global function field. The first step (Algorithm 18) is to calculate the length locally (i.e., with respect to a given place). Then, one can use a local-global principle to compute the length globally. This is done in Algorithm 19. The remaining algorithms use similar arguments to calculate other important numbers, namely the level, the Pythagoras number and a Pythagoras element for a global function field.

A nice feature in this dissertation is that for each problem, the author presents examples that illustrate each step of the corresponding algorithms.

The results presented in this dissertation are important and the algorithms developed consist of a valuable contribution to computational algebra. The text shows that the author has broad knowledge of many areas of mathematics. Moreover, the results in this dissertation have strong potential for continuation and improvements. Because of this, I recommend it to be accepted.

The submitted dissertation fulfils the conditions set by Article 187 ust. 1-3 ustawy z dnia 20 lipca 2018 r. Prawo o szkolnictwie wyższym i nauce (Dz. U. z 2022 r. poz. 574 ze zm.)