

Abstract

This dissertation presents computational tools for quadratic forms over global function fields of characteristic different from 2. The majority of the algorithms we develop in this study rely on our ability to find all places dividing any coefficient of the given quadratic form. This problem is equivalent to the factorization of fractional ideals in the ring of polynomial functions of a global function field. As a result, we begin by presenting alternative approaches for factoring fractional ideals that do not rely on determining the maximum order of the global function field in question. We then propose techniques for tackling the following quadratic form theory computational problems: how to detect whether a quadratic form is isotropic or not, how to detect whether a quadratic form is hyperbolic or not, how to compute the anisotropic dimension (or equivalently the Witt index) of a quadratic form, how to construct an anisotropic part of a quadratic form, how to determine if two forms are Witt-similar or are Ono-similar or not. We further explore algorithms for computing some important field invariants that are linked to quadratic forms. Those are: the length of a sum of squares, the level of a field, the Pythagoras number of a field, as well as a Pythagoras element of a field.

The research work that constitutes the basis of this doctoral thesis has been published in the following series of publications:

1. Mawunyo Kofi Darkey-Mensah. 2022. Algorithms for quadratic forms over global function fields of odd characteristic. *ACM Commun. Comput. Algebra*. 55, 3 (September 2021), 68–72.
2. Mawunyo Kofi Darkey-Mensah and Beata Rothkegel. 2021. Computing the Length of Sum of Squares and Pythagoras Element in a Global Field. *Fundam. Inf.* 184, 4 (2021), 297–306.
3. Mawunyo Kofi Darkey-Mensah, Przemysław Koprowski, and Beata Rothkegel. 2021. The Anisotropic Part of a Quadratic Form over a Global Function Field. In *Proceedings of the 2021 on International Symposium on Symbolic and Algebraic Computation (ISSAC '21)*. Association for Computing Machinery, New York, NY, USA, 115–122.
4. Mawunyo Kofi Darkey-Mensah and Przemysław Koprowski. 2019. Intrinsic Factorization of Ideals in Dedekind Domains. *Fundam. Inf.* 170, 4 (2019), 325–338.